## Cambridge International AS \& A Level

## MATHEMATICS <br> 9709/62 <br> Paper 6 Probability \& Statistics 2 <br> May/June 2021 <br> MARK SCHEME

Maximum Mark: 50

## Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.
Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE ${ }^{\text {M }}$, Cambridge International A and AS Level components and some Cambridge O Level components.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

## GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.


## GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

## GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.


## GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

## GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

## GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles
1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3
Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5 Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.

6
Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

## Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.
DM or DB When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.


## Abbreviations

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO Correct Working Only
ISW Ignore Subsequent Working
SOI Seen Or Implied
SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1(a) | $\begin{aligned} & \mathrm{H}_{0}: p=\frac{1}{4} \\ & \mathrm{H}_{1}: p \neq \frac{1}{4} \end{aligned}$ | B1 | or $\mathrm{H}_{0}: \mu=25$ <br> or $\mathrm{H}_{1}: \mu \neq 25$ |
|  |  | 1 |  |
| 1(b) | $\mathrm{N}\left(25, \frac{75}{4}\right)$ | B1 | SOI. <br> Allow B1 for $\mathrm{N}\left(25, \frac{75}{4}\right)$ or $\mathrm{N}(0.25,0.001875)$ SOI. |
|  | $\pm \frac{15.5-25}{\sqrt{\frac{75}{4}}} \text { or } \frac{\frac{15.5}{100}-0.25}{\sqrt{\frac{0.25 \times 0.75}{100}}}$ | M1 | Standardise with their $\mathrm{N}(25, \ldots)$ <br> Allow with no or wrong continuity correction. |
|  | $\pm$-2.194 (2.19) | A1 |  |
|  | $-2.326<-2.194$ or $0.0141>0.01$ or $0.9859<0.99$ | M1 | For valid comparison (accept 2.326 to 2.329) |
|  | No evidence to reject that the probability is $\frac{1}{4}$ | A1 FT | OE must be in context and not definite, e.g. not 'Claim untrue'. No contradictions. <br> FT their $z$; dependent on two-tailed test (one-tailed test can score B1 M1 A1 M1 A0) <br> SC for use of Binomial $\mathrm{B}(100,0.25) \mathrm{P}=0.0111$ for B 1 and then comparison with 0.01 and correct conclusion for B 1 , maximum 2 out of 5 marks. |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 2(a) | $\operatorname{Var}(X)=400 \times 0.01 \times 0.99(=3.96)$ | M1 |  |
|  | $\operatorname{Var}(4 X+2)=16 \times \operatorname{Var}(X)$ | M1 | For $16 \times$ their $\operatorname{Var}(X)$ |
|  | 63.36 | A1 | Accept 63.4 |
|  |  | 3 |  |
| 2(b)(i) | $\mathrm{Po}(4)$ | B1 |  |
|  | $n=400>50$ and either $n p=4<5$ or $p=0.01<0.1$ | B1 | Must quote values 400 and 4 or clearly see $n=400$ and $n p=4$ (or $p=0.01$ ) in working |
|  |  | 2 |  |
| 2(b)(ii) | $e^{-4}\left(\frac{4^{2}}{2!}+\frac{4^{3}}{3!}+\frac{4^{4}}{4!}+\frac{4^{5}}{5!}\right)$ | M1 | FT their '4' <br> Allow one end error <br> FT from (b)(i) Use of Normal allow M1 for attempt at standardising (with correct continuity correction) using their $\mathrm{N}(4,3.96)$ and attempt at probability. <br> FT from (b)(i) Use of Binomial allow M1 for attempt at $\mathrm{P}(2,3,4,5)$ Binomial terms clearly seen and added |
|  | 0.694 (3 sf) | A1 | CWO <br> SCB1 only for unsupported answer of 0.694 |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 3(a) | $\frac{1}{2} p(p-1)=1$ | M1 | For area $=1$ <br> For verification methods accept $\frac{1}{2} \times 2 \times 1=1$ or $\frac{1}{2} \times 2 \times(p-1)=1$ or $\frac{1}{2} \times 1 \times p=1$ as indication that area $=1$ |
|  | $p=2$ | A1 | AG - Convincing method and answer. Must see quadratic rearranged to $=0$ and no errors seen. N.B. Accept convincing verification methods (e.g. statement such as 'assume $p=2$ ' or 'if $p=2$ ' or 'using $p=2$ ' or showing by clear substitution that $p=2$ fits $\frac{1}{2} p(p-1)=1$ with clear conclusion) |
|  |  | 2 |  |
| 3(b) | Gradient $=2$ <br> equation of line is $y=2 x+c$ <br> line passes through $(1,0)$, hence $c=-2$ | M1 | Award for attempting equation of line $y=m x+c$ with $m=2,-2$, $\frac{1}{2}$ or $-\frac{1}{2}$ and numerical $c(c \neq 0)$ |
|  | $y=2 x-2$ | A1 | May be seen in (a) <br> M1 can be implied by correct answer |
|  | $2 \int_{1}^{2}\left(x^{2}-x\right) \mathrm{d} x$ | M1 | For attempting $\int x f(x) \mathrm{d} x$. Ignore limits, FT their equation. |
|  | $2\left[\frac{x^{3}}{3}-\frac{x^{2}}{2}\right]_{1}^{2}$ | A1 FT | Correct integration FT their $\mathrm{f}(x)$ and correct limits |
|  | $\frac{5}{3} \text { or } 1.67(3 \mathrm{sf})$ | A1 |  |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 4(a) | Mean $=15.0+32.0+8.6[=55.6]$ | B1 | Allow unsimplified |
|  | $\operatorname{Var}=1.1^{2}+3.5{ }^{2}+1.2^{2}[=14.9]$ | B1 | Allow unsimplified |
|  |  | 2 |  |
| 4(b) | $\frac{60-" 55.6 "}{\sqrt{14.9 "}}[=1.140]$ | M1 | FT their 55.6 and 14.9 Ignore continuity correction |
|  | $1-\phi(" 1.140$ ") | M1 | For correct probability area consistent with their working |
|  | 0.127 (3 sf) | A1 | CWO |
|  |  | 3 |  |
| 4(c) | $\frac{54.5-" 55.6^{"}}{\sqrt{\frac{" 14.9^{"}}{15}}} \text { or } \frac{817.5-834}{\sqrt{223.5}}[=-1.104]$ | M1 | FT their 55.6 and 14.9 No mixed methods |
|  | 1-ф("1.104") | M1 | For correct probability area consistent with their working |
|  | 0.135 (3 sf) | A1 | As final answer |
|  |  | 3 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 5(a) | Conclude that (population) mean time has changed (or is not 42.4) although $\mu$ has not changed (or is still 42.4) | B1 | OE. In context. |
|  |  | 1 |  |
| 5(b) | $\mathrm{H}_{0}$ : population mean $($ or $\mu)=42.4$ $\mathrm{H}_{1}$ : population mean $($ or $\mu) \neq 42.4$ | B1 | Not just 'mean'. (could be seen in (a)) |
|  | $\pm \frac{45.6-42.4}{\sqrt{38.2 \div 20}}$ | M1 | For standardising (must have $\sqrt{20}$ ) |
|  | $\pm 2.315$ | A1 |  |
|  | $2.240<{ }^{\prime} 2.315$ ' | M1 | For valid comparison (accept 2.241) or $\mathrm{P}(z>2.315)=0.0103<0.0125$ oe |
|  | There is evidence that $\mu$ or mean time has changed | A1 FT | FT their z <br> In context, not definite. No contradictions. <br> Note: Accept correct alternative methods <br> SC: One tail test no FT. Can score B0 M1 A1 M1 (comparison with 1.96) A0 (maximum 3 out of 5) |
|  |  | 5 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 6(a) | $\text { est }(\mu)=\frac{7570}{100}(=75.7)$ | B1 |  |
|  | $\begin{aligned} & \operatorname{est}\left(\sigma^{2}\right)=\frac{100}{99}\left(\frac{\sum h^{2}}{100}-75.7^{\prime 2}\right) \text { or } \frac{1}{99}\left(588050-\frac{7570^{2}}{100}\right) \\ & =\frac{100}{99}\left(\frac{588050}{100}-{ }^{\prime} 75.7^{12}\right)[=151.525] \end{aligned}$ | M1 | Attempted <br> (Note: Biased variance (150.01) scores M0 ) |
|  | $=152(3 \mathrm{sf})$ | A1 | $\text { Or } \frac{15001}{99}$ |
|  |  | 3 |  |
| 6(b) | $\cdot 75.7 ’ \pm z \sqrt{\frac{151.525}{100}}$ | M1 | For expression of correct form. Must be a $z$ value. Condone just + or just -. |
|  | $z=2.576$ | B1 | Accept 2.574 to 2.579 |
|  | 72.5 to 78.9 | A1 FT | FT biased variance only Must be an interval |
|  |  | 3 |  |
| 6(c) | $0.99^{4}$ | B1 |  |
|  | 0.961 (3 sf) | B1 |  |
|  |  | 2 |  |


| Question | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 7(a) | $\mathrm{e}^{-4.2} \times \frac{4.2^{4}}{4!}$ | M1 | $\mathrm{P}(4)$, allow any $\lambda$ |
|  | 0.194 (3 sf) | A1 | As final answer. SC Unsupported correct answer scores B1 only. |
|  |  | 2 |  |
| 7(b) | $1-\mathrm{e}^{-8.4}\left(1+8.4+\frac{8.4^{2}}{2}+\frac{8.4^{3}}{3!}\right)$ | M1 | Allow M1 with incorrect $\lambda$. Accept one end error. |
|  | 0.968 (3 sf) | A1 | As final answer. <br> SC Unsupported correct answer scores B1 only. |
|  |  | 2 |  |
| 7(c) | $\mathrm{N}(50.4,50.4)$ | M1 | SOI |
|  | $\frac{39.5-50.4}{\sqrt{50.4}} \quad[=-1.535]$ | M1 | Allow wrong or no continuity correction. Must have $\sqrt{ }$ |
|  | $\Phi\left({ }^{`}-1.535{ }^{\prime}\right)=1-\Phi\left({ }^{\prime} 1.535{ }^{\prime}\right)$ | M1 | For correct probability area consistent with their working. |
|  | $0.0624(3 \mathrm{sf})$ or 0.0623 | A1 |  |
|  |  | 4 |  |

